Paper: Thermal and Statistical Physics and Mathematical Methods in Physics

Time: $2\frac{1}{2}$ hrs. M. M.: 75

N.B.:

1. All questions are compulsory.
2. Figures to the right indicate full marks.
3. Draw neat diagrams wherever necessary.
4. Symbols have usual meaning unless otherwise stated.
5. Use of non-programmable calculator is allowed.

1. (a) Attempt any **one: -**

   (i) Define Random Variable. State and explain the probability function for the random variable x. **10**

   (ii) State and prove the probability theorem of the two successive events A and B. **10**

   (b) Attempt any **one: -**

   (i) A club consisting of 40 members. In how many ways a president, Vice president, secretary and treasurer can be chosen? In how many ways a committees of 4 members chosen? **5**

   (ii) A random variable x takes the value 0,1,2,3 with probabilities $\frac{5}{12}, \frac{1}{3}, \frac{1}{12}, \frac{1}{6}$ compute the mean and standard variation for x? **5**

2. (a) Attempt any **one: -**

   (i) Explain terminology and notation of a complex number z. Find $x, y. r, \theta$ of given complex number z. Plot the number and label it and find its complex conjugate. **10**

   (a) $z = \sqrt{2}e^{-i\pi/4}$,
   (b) $z = -1$

   (ii) Explain logarithm of a complex numbers and Find the principle value of **10**

   (a) $\ln(-1)$
   (b) $\ln\left(\frac{1+i}{1-i}\right)$

   (b) Attempt any **one: -**

(i) If $f(z) = \frac{z^2+1}{z-3}$ then find $f(i-2)$ **5**

(ii) A particle moves in (x, y) plane so that its position (x, y) as a function of time t is given by **5**

$$z = 1 + 3e^{2it}$$

Calculate the magnitude of velocity and acceleration.

3. (a) Attempt any **one: -**

(i) Define Canonical Ensemble. State expression for Energy and Weight for it. How canonical partition function Q is related with q for an ideal gas? How does this relationship differ for distinguishable and indistinguishable particles? **10**

(ii) What is entropy? Derive the Boltzmann formula $dS = k\, d(lnW)$. **10**

(b) Attempt any **one: -**

(i) Prove that the total energy E for canonical ensemble is given by **5**

$$E = NkT^2 \frac{dlnq}{dT}$$

(ii) Evaluate the translational partition function for Ar confined to a volume of 1000 cm3 at 298 K. At what temperature will the translational partition function of Ne be identical to that of Ar at 298 K confined to the same volume? **5**

Given: Mass of Ne = 3.35×10^{-26} Kg

Mass of Ar = 6.63×10^{-26} Kg

4. (a) Attempt any **one: -**

(i) Derive the Fermi–Dirac distribution law that describes the occupancy of energy states by fermions in thermal equilibrium. **10**

(ii) Derive the Rayleigh–Jeans formula for the spectral energy density of blackbody radiation and explain its limitation at high frequencies. **10**

(b) Attempt any **one: -**

(i) Derive the expression for the root mean square (RMS) speed of gas molecules using the Maxwell–Boltzmann distribution. **5**

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(ii) Calculate the most probable speed, average speed and rms speed of CO_2 molecules at 400K. Mass of a CO_2 molecule is 44 X 10^{-3} kg/mole, 5

R = 8.314 J/K, $N_A = 6 \times 10^{23}$.

5. Attempt any **Five: -**

(i) Define and explain mean value , variance and standard deviation 3

(ii) A random variable x takes the value 0,1,2,3 with probabilities $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$. 3

Calculate the mean for x?

(iii) Evaluate: $\cos(\pi + i \ln 5)$ 3

$(Use: e^{\pm \pi i} = -1)$

(iv) Express $\left(2e^{i2\pi/3}\right)^{10}$ in $x + iy$ form. 3

(v) What is the value of partition function for molecule having energy levels $\varepsilon_1 = 0, \varepsilon_2 = 500\ cm^{-1} and\ \varepsilon_2 = 1500\ cm^{-1}$ with degeneracies 1, 3 and 5 respectively at T = 300 K? (Take $k = 0.6950\ cm^{-1}K^{-1}$) 3

(vi) What is the weight associated with the configuration corresponding to observing 40 heads after flipping a coin 100 times? How does this weight compare to that of the most probable outcome? 3

(vii) Calculate the average velocity of a particle in a gas whose density is 2.42 kg / m^3 at a pressure of 10^5 N / m^2. 3

(viii) Temperature of a cubical black body cavity of edge 5 cm is 2000 K. calculate number of modes of vibration per unit volume in the cavity between wavelengths 4000 Å and 4010 Å. 3

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Constants:

Charge of an electron e = 1.6021×10^{-19} Coulomb, Mass of an electron $m_e = 9.11 \times 10^{-31}$ kg
Boltzmann constant $k_B = 1.38054 \times 10^{-23}$ Joule/Kelvin, Planck's constant $h = 6.626 \times 10^{-34}$ joule sec
Permeability of free space $\mu_0 = 4 \times 10^{-7}$ Henry/m, Avogadro's number NA = 6.023×10^{26} /kg mole

1. (a) Attempt any **one: -**

(i) Explain the basic electrostatic properties of ideal conductors. **10**

(ii) Suppose a point charge **q** is held a distance **d** above an infinite grounded conducting plane. Show that the total charge induced on the plane is **−q** based on the classic image problem. **10**

(b) Attempt any **one: -**

(i) Show that $\nabla \cdot E = \frac{\rho}{\varepsilon_0}$, symbols with usual meaning. **5**

(ii) Determine electric field due to potential $V = x^2 + 2y^3 + 3z^4$. **5**

2. (a) Attempt any **one: -**

(i) A dipole $\overline{p}$ is placed in an uniform electric field $\overline{E}$, show that the net torque acting on the dipole is $\overline{N} = \overline{p} \times \overline{E}$. Derive an expression for the incremental force and net torque acting on the dipole placed in a non-uniform electric field $\overline{E}$. **10**

(ii) Derive expressions for divergence of magnetic field $\vec{B}$ and discuss its physical significance. **10**

(b) Attempt any **one: -**

(i) Starting from $\overline{\nabla} x \overline{B} = \mu_0 \overline{J}$, find out the magnitude of magnetic field at a perpendicular distance 'r' from a long straight wire carrying current 'I'. **5**

(ii) A vector field in vacuum is given by $\overline{B} = 2yz\,\hat{x} + 4xz\,\hat{y} + 6xy\,\hat{z}$. Show that it could be a magnetostatic field. Find current density in the field at (3, 4, -5). **5**

3. (a) Attempt any **one:-**

(i) Obtain the boundary conditions to be satisfied by **E, D, B** and **H** fields at the interface between two medium. **10**

(ii) Derive Maxwell's equation inside the matter. **10**

(b) Attempt any **one:-**

(i) Obtain an expression for Torques on Magnetic dipoles in a Magnetic field. **5**

(ii) A long copper rod of radius **R** carries uniformly distributed free current I, find the auxiliary field $\vec{H}$ inside the rod. **5**

4. (a) Attempt any **one:-**

(i) Show that $\frac{dw}{dt} = -\frac{\partial}{\partial t}\int_v (u_e + u_m)\, d\tau - \oint_s \vec{S}.\hat{n}da$. Symbols have usual meaning. **10**

(ii) Derive expressions for time average of energy density $<u>$, Poynting vector $<\vec{S}>$ and momentum density $<\vec{P}>$ for plane electromagnetic wave travelling along z – axis. **10**

(b) Attempt any **one:-**

(i) Obtain equation of continuity from the principle of conservation of charge. **5**

(ii) Calculate the amplitude of electric field in the laser beam in vacuum with diameter of 2mm and power 4mW.Given $\sqrt{\frac{\mu_0}{\varepsilon_0}}=377\Omega$ **5**

5. Attempt any **Five:-**

(i) Find the energy of a uniformly charged spherical shell of total charge $4\mu C$. **3**

(ii) Find the work done to a move a charge of 3 Coulombs under the influence of potential difference 4 Volts. **3**

(iii) A long solenoid consists of 5000 turns per meter of its length. What must be the steady current flowing through the windings of the solenoid to produce a magnetic field of 5×10^{-3} Wb/m^2? Given $\mu_o = 4\pi \times 10^{-7}$ N/A^2. **3**

(iv) Compute the Polarisability of Helium at STP in an external electric field 6000 V/m. Given: Atomic polarisability- 0.205×10^{-30} m^3. Number of helium atoms per unit volume is 0.269×10^{26} per K-mole. **3**

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(v) Write down the basic equations of Electrodynamics, specifying the divergence and the curl of electric and magnetic fields. 3

(vi) Show that the divergence of the bound volume current density is zero. 3

(vii) If $\bar{E} = E_0\, e^{i(kz-wt)}\hat{x}$ and $\bar{B} = B_0\, e^{i(kz-wt)}\hat{y}$. Find the energy per unit time per unit area transported by the fields in free space. 3

(viii) An electromagnetic wave is incident normally on the surface of water with refractive index 1.33 from air. Calculate the percentage of incident intensity transmitted from air. 3

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1. (a) Attempt any **one: -**

(i) Using variable separation method for Schrodinger's partial differential equation for hydrogen atom, obtain the three ordinary differential equations in terms of R, Θ, Φ. **10**

(ii) What are symmetric and anti-symmetric wave functions? Show that system of electrons is described by anti-symmetric wave functions. **10**

(b) Attempt any **one: -**

(i) Calculate the most probable value of electron's radial coordinate, using ground state wave function of hydrogen atom $\Psi = \frac{e^{-r/a_0}}{{a_0}^{3/2}.\pi^{1/2}}$ where a_0 is the Bohr's radius. **5**

(ii) Draw the space quantization diagram of orbital angular momentum for a d-electron. Calculate cosine of the angle between each orientation of $\vec{L}$ and Z – axis. **5**

2. (a) Attempt any **one: -**

(i) Prove that an electron undergoing transition from higher energy level E_m to lower energy level E_n, emits radiation of frequency $\upsilon = \frac{E_m - E_n}{h}$, according to quantum mechanics. **10**

(ii) State Zeeman effect. Write the quantum mechanical explanation of Normal Zeeman effect. **10**

(b) Attempt any **one: -**

(i) Discuss L-S coupling scheme for two- electron atom, using a suitable vector diagram. **5**

(ii) A sample of certain element is placed in a magnetic field of 0.3 Wb/m^2? What will be the spacing between the normal Zeeman components of a spectral line of wavelength 4500 AU? **5**
Given: $c = 3 \times 10^8$ m/s , $e = 1.6 \times 10^{-19}$ C, $m = 9.1 \times 10^{-31}$ Kg

3. (a) Attempt any **one:-**

(i) Assuming a diatomic molecule to be a rigid rotator, derive an expression for its moment of inertia. Hence, find an expression for its rotational energy and sketch the energy level diagram. **10**

(ii) Write expression for vibration-rotation energy of a diatomic molecule using Born Oppenheimer approximation. Find the expression for the frequency of spectral lines in P and R branch of the spectrum. Also draw vibration-rotation spectrum of diatomic molecule. **10**

(b) Attempt any **one:-**

(i) Draw a schematic diagram of an absorption IR spectrometer. Also explain its parts in short. **5**

(ii) The masses of mercury and chlorine atoms are 33.2×10^{-26} Kg and 5.81×10^{-26} Kg respectively. The bond length of HgCl is 2.23×10^{-10} m. Calculate reduced mass and the moment of inertia of HgCl molecule. **5**

4. (a) Attempt any **one:-**

(i) What is the basic principle of Nuclear Magnetic Resonance? Explain with neat diagram Nuclear Magnetic Resonance (NMR) spectrometer. **10**

(ii) Discuss pure rotational Raman spectra of linear molecules and show that stokes and anti-Stokes lines are symmetrically placed about the parent line. **10**

(b) Attempt any **one:-**

(i) List five observations based on Raman Effect. **5**

(ii) A Stokes line of wavelength 5000 A was observed in a Raman spectrum when the radiation of wavelength 4800 A was scattered by a medium. Calculate the Raman frequency of the corresponding Anti-Stokes line. **5**
Given : $c = 3 \times10^{8}$ m/s

5. Attempt any **Five:-**

(i) Solve the Φ – equation for hydrogen atom and normalize the wave function. Name the quantum number introduced. **3**

(ii) Write L_z in spherical polar co-ordinates and obtain its eigen value. **3**

(iii) Calculate the energy difference between the split states due to spin-orbit interaction if the magnetic field at the site of the electron due to its orbital motion is 14 Wb/m^2. **3**
Given: Bohr magneton $\mu_B = 9.274 \times 10^{-24}$ J/T

(iv) Find the value of Lande's g-factor for ^3P and ^5D states. **3**

(v) The rotational absorption for J = 0 to J = 1 occurs at 1.153×10^{11} Hz in $C^{12}O^{16}$ and at 1.102×10^{11} Hz in C^xO^{16}. Calculate 'x' .Assume bond length to be the same in both molecules. **3**

(vi) The frequency of oscillations of atoms in a CO molecule is 3×10^{13}Hz. Calculate the force constant. **3**
Given: $M(C^{12}) = 1.99 \times 10^{-26}$ kg , $M(O^{16}) = 2.66 \times 10^{-26}$ kg .

(vii) A particular Nuclear Magnetic Resonance instrument operates at 100 MHz, calculate magnetic field required to bring ^{13}C nuclei to resonance. **3**
Given: g- factor for ^{13}C nuclei $g_I = 1.404$
Planck's constant $h = 6.62 \times10^{-34}$ J.s,
Nuclear magneton $\mu_N = 5.05\times10^{-27}$ J/T

(viii) Calculate the Electron spin resonance (ESR) frequency for free electrons required at a magnetic field strength of 2 T **3**
(Given g – factor = 2.002, Bohr magneton $\mu_B = 9.274 \times 10^{-24}$ J/T, Planck's constant $h = 6.63 \times 10^{-34}$ J.s)

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Time: 2 1/2 hrs. **M. M.: 75**

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Constants: Boltzmann Constant: $k_B = 1.38054 \times 10^{-23}$ Joule/Kelvin
Planck's Constants: $h = 6.626 \times 10^{-34}$ joule sec

1. (a) Attempt any **one: -**
   - (i) What are Miller indices? With example, explain how they are obtained? Sketch the (0 1 0), (1 1 0) and (1 1 1) planes in a cubic crystal. **10**
   - (ii) Explain the fourteen three dimensional space lattice with the help of a neat diagram. Give in detail relation between axes of symmetry and corresponding angles. **10**

   (b) Attempt any **one: -**
   - (i) Define the terms: Coordination number, Nearest neighbor distance, Atomic radius, and Atomic packing factor. **5**
   - (ii) Calculate the radius of atoms in α-iron belonging to bcc structure. Take the density of α-iron as 7860 kg/m^3. Atomic weight of iron is 55.85. **5**

2. (a) Attempt any **one: -**
   - (i) Based on quantum mechanical considerations, obtain the expression for electrical conductivity. Hence show that resistivity of metals is directly proportional to the absolute temperature. **10**
   - (ii) Define Fermi Energy. State the expression for density of energy states Z(E) dE in metals and obtain the expression for fermi energy of metals at absolute zero in terms of electrons per unit volume of the metal. **10**

   (b) Attempt any **one: -**
   - (i) Show that the probability that a state ΔE above the Fermi level E_F is filled equals the probability that a state ΔE below E_F is empty. **5**
   - (ii) Calculate the probability that an allowed state occupied by an electron lies above the fermi level by K_BT, 2 K_BT. **5**

3. (a) Attempt any **one: -**
   - (i) With the help of neat diagram derive the expression for the effective mass of an electron in a crystal. Also discuss its variation in an energy band. **10**
   - (ii) Explain the concept of carrier life time. Set up the continuity equation for the charge carrier in a semiconductor. **10**

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(b) Attempt any **one: -**

(i) Write a short note on Brillouin zones. **5**

(ii) The minority carrier lifetime in p-type material is 10^{-7} s. The mobility of electron in silicon is $\mu_n = 0.15\ m^2/Vs$. Find the diffusion length of Si at 300K. **5**
Given: $k_B = 1.3805 \times 10^{-23}$Joule/kelvin, e $= 1.6021 \times 10^{-19}$C.

4. (a) Attempt any **one: -**

(i) Write a detailed note on London equation in case of superconductors, explaining the penetration depth. **10**

(ii) Explain the Band Structure of an open circuit p-n junction, discussing the energy levels of the different bands. **10**

(b) Attempt any **one: -**

(i) Explain the volt-ampere characteristics of the p-n junction diode. **5**

(ii) A superconducting tin has a critical temperature of 3.7 K at zero magnetic field and a critical field of 0.0306 Tesla at 0 K. Find the critical field at 2 K. **5**

5. Attempt any **Five: -**

(i) Copper has fcc structure and its atomic radius is 0.1278 nm. Calculate its density. Take the atomic weight of copper as 63.5. **3**

(ii) Zinc has hcp structure. The height of the unit cell is 0.394 nm. The nearest neighbor distance is 0.28 nm. Calculate the volume of the unit cell. **3**

(iii) Fermi energy of electrons in copper is 7.047 eV. Calculate the fermi velocity of electrons in copper. **3**

(iv) For copper, $\sigma = 6 \times 10^7$mhos/m, n $= 8.47 \times 10^{28}/m^3$. Calculate the relaxation time. **3**

(v) Consider a two-dimensional square lattice of side 0.3 nm. At what electron momentum values do the sides of the first Brillouin zone come? (Given: h $= 6.626 \times 10^{-34}$Js, m $= 9.11 \times 10^{-31}$ kg). **3**

(vi) If Hall coefficient of n-type semiconductor a sample is -0.0125m^3 /C. Determine the carrier density in the sample. **3**

(vii) A germanium p-n junction has reverse saturation current, $I_0 = 2\ \mu$A at 27^0C. Find its static resistance for an applied forward bias of 0.2 V at 27^0C. **3**

(viii) Calculate critical current through Tungsten wire of diameter 3 mm and $H_c = 8.51 \times 10^7$ A/m. **3**

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